Postdoctoral fellowship: Neural networks and tropical geometry

Advisors: Mateusz Skomra (LAAS-CNRS)
Georg Loho (FU Berlin)

Context The *rectified linear unit* (ReLU) is one of the most prominent activation functions used in neural networks. Contrary to other popular activation functions, ReLU is a piecewise affine function that is not differentiable at all points of its domain. In particular, one can study neural networks with this activation function using tools from discrete geometry additionally to classical more analytic methods. As a step in this direction, it was recently discovered that neural networks with ReLU activation function are deeply connected to the area of *tropical geometry*, which replaces the study of polynomials with the study of piecewise affine functions.

This has already lead to new advances on the decision boundary of neural networks [ZNL18], to bound the number of linear regions of functions represented by these networks [MRZ22], to study their expressivity [HHL23], and to certify their robustness [GPP$^+$21], to give a few examples.

Assignment The research project will aim to explore the connection between discrete / tropical geometry and machine learning. A non-exhaustive list of potential topics include:

- exploring neural networks as models of computation extending [HS25, HL24],
- static analysis and robust certification of ReLU neural networks extending [GPP$^+$21],
- studying geometric objects associated with ReLU neural networks with tools from tropical geometry extending [ZNL18, BLM24].

Main activities The position will be funded through the chair "Combining Polynomial Optimization and Machine Learning (POPML4PS)", part of the ANITI AI cluster. The main advisors for this position are Georg Loho (FU Berlin) and Mateusz Skomra (LAAS-CNRS). The research will be achieved by working with Mateusz Skomra and other members of the chair at LAAS-CNRS as well as visits to the group of Georg Loho. The position is equipped with ample travel funding.

Application requirements A successful candidate will have a strong background in tropical geometry or mathematical foundations of machine learning as well as basic programming skills. The applicant should hold a PhD degree in mathematics or computer science, or a related field, and have an excellent track record of publications in tropical / discrete geometry, machine learning or related areas of computer science.

The candidates are kindly asked to send an email to Georg Loho and Mateusz Skomra, georg.loho@math.fu-berlin.de, mateusz.skomra@laas.fr, with "Postdoc candidate" in the title, a full CV with a detailed publication track record, a motivation letter (max. 2 pages), as well as names of people who could provide a reference letter. Knowledge of French does not constitute a pre-requisite, but an excellent command of English is demanded.

Working place and salary The postdoctoral position is expected to start at any date after May 2026. The postdoctoral fellow will be hosted at LAAS-CNRS in Toulouse, within the POP team lead by Victor Magron. The remuneration is according to the candidate profile. The contract duration is 12 months, with a possible extension for another 12 months. This is a full time research position, with no teaching duties.

References

[BLM24] Marie-Charlotte Brandenburg, Georg Loho, and Guido Montufar. The real tropical geometry of neural networks for binary classification. *Transactions on Machine Learning Research*, 2024.

[GPP+21] Eric Goubault, Sébastien Palumby, Sylvie Putot, Louis Rustenholz, and Sriram Sankaranarayanan. Static analysis of ReLU neural networks with tropical polyhedra. In *Proceedings of the 28th Static Analysis Symposium (SAS)*, pages 166–190, 2021.

[HHL23] Christian Haase, Christoph Hertrich, and Georg Loho. Lower bounds on the depth of integral relu neural networks via lattice polytopes. In *Proceedings of the 11th International Conference on Learning Representations (ICLR)*, 2023.

[HL24] Christoph Hertrich and Georg Loho. Neural networks and (virtual) extended formulations. *CoRR*, abs/2411.03006, 2024.

[HS25] Christoph Hertrich and Leon Sering. Relu neural networks of polynomial size for exact maximum flow computation. *Math. Program.*, 210(1):377–406, 2025.

[MRZ22] Guido Montúfar, Yue Ren, and Leon Zhang. Sharp bounds for the number of regions of maxout networks and vertices of minkowski sums. *SIAM Journal on Applied Algebra and Geometry*, 6(4):618–649, 2022.

[ZNL18] Liwen Zhang, Gregory Naitzat, and Lek-Heng Lim. Tropical geometry of deep neural networks. In *Proceedings of the 35th International Conference on Machine Learning (ICML)*, pages 5824–5832, 2018.