Combining Math and Sustainability for Teaching

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joint with

- Benedikt Weygandt (Didactics of Mathematics): didactic input
- Sofia Garzon Mora (PhD student in Discrete Algebraic Geometry with Christian Haase): support in details of math exercises

Earlier attempts with this subject

- earlier attempts to combine math and sustainability: joint with Timo de Wolff gathering interested early career mathematicians
- Climathics Workshop December 2022
- Winter Semester 23/24: Seminar Math and Sustainability

conclusions:

- advanced mathematical basics necessary for actual understanding
- actually very motivating topic
- requires a lot of work to bridge the gap

Sustainability

Clarify:

- SDGs
- Planetary boundaries (also societal needs)
- ...

e.g. climate change, decline of ecosystems, social aspects, ...

Combination of math and sustainability

- aspects of **sustainability** in addition to math
- **extra motivation** through choice of relevant topic

Which math?

Where does math come in?

- analyzing the status
- interpolating into the future
- exploring solutions

Idea:

- *Who*? – individual vs collective ∼ game theory
- *When*? – time dependence ∼ dynamical systems
- *What*? – different goals ∼ optimization
- *How*? – uncertainty ∼ data & probability

What the course tries to be (as a math course)

- spectrum between pure math, applied math, toy applications, other fields using mathematical tools
- not a modeling course
- mainly structures, barely solution techniques

Goals

. . . as a math course

- emphasis on **relations between the mathematical subjects**
- learn to work with **sources beyond knowledge**, finding and assessing references

. . . as a modern math course

- convey mathematical knowledge but also **further skills**
- **participative aspect**: choosing focus topics by themselves, forming opinion about ones own interests
- **working in groups** for several advanced tasks

Mathematical goals

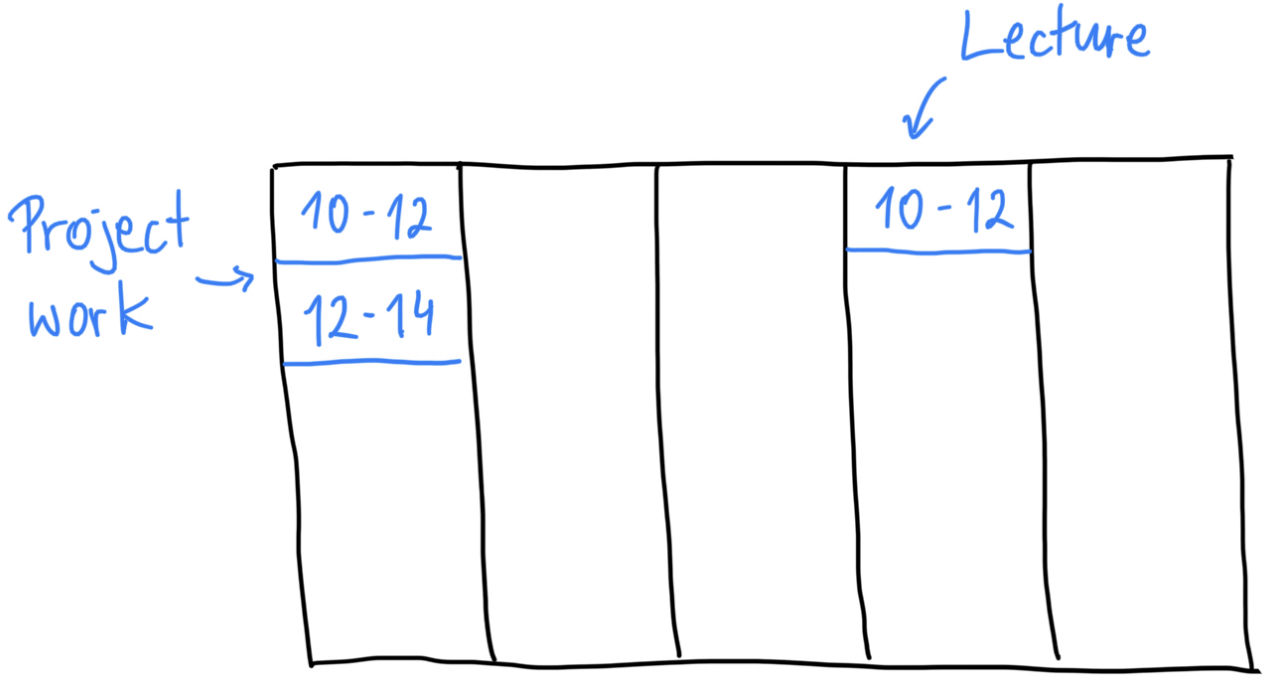
- give broad **overview** of some areas of math: optimization, game theory, data & probability, dynamical systems
- exhibit and explore **similarities** in different areas of mathematics *(e.g. Pareto partial order in game theory and multicriteria optimization)*
- highlight on a mathematical level the crucial **aspects behind sustainability** *(several objectives, several players, time dependence, uncertainty)*

My preparation

Talking to experts!

- Boris Zimmermann (e-mobility, logistics)
- Rupert Klein (fluid dynamics, research around climate change)
- Heinrich Nax (game theory)
- Jobst Heitzig (climate / economic / societal models)
- Laurin Köhler-Schindler (probability theory)
- ...

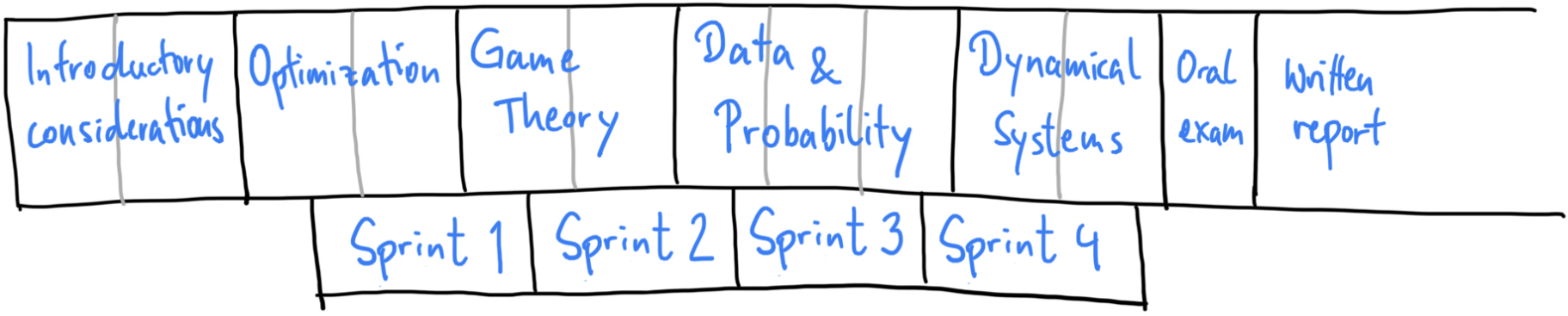
Structure of the course



- **Lectures**: providing mathematical foundations
- **Project time**: for solving Challenges (Exercises on basic notions) and working on Sprints

Note: all work in groups of three

Structure of the course



About **Sprints**

- have to cover at least 3 out of the 4 subject areas
- self-chosen topic from a long list clustered in 3 types (wicked problems, specific applications, related mathematical topics)

Lectures: content (1, 2)

Introductory

- what is sustainability
- what are wicked problems
- what is modeling
- what is systems thinking

Lectures: content (3, 4)

Optimization: *basic optimization problems, optimality conditions, very few (fundamental) algorithms*

Lectures: content (5, 6)

Game Theory: *standard forms of games, several examples of games with relation to societal conflicts, equilibria notions, fixed points*

Lectures: content (7, 8, 9)

Data & Probability: *recap of very basic notions, data presentation, probability inequalities, limit theorems, stochastic processes, Markov chains*

Lectures: content (10, 11)

Dynamical Systems: *discrete and continuous dynamical systems, fixed points / critical points, bifurcation, limit behaviour, solving differential equations (analytic, numerical, using software)*

Tasks 1: Challenges

Challenges (per subject area)

- close to the definitions
- only pass or fail
- 1 – 2 exercises with several parts

Tasks 2: Sprints

- choosing topics from a long list of options (see next slide)
- gather findings in Miro-Board
- goal: short presentation for the other students including feedback (idea: become expert on one question, learn also from findings of the other students)

Topics

Three types

- wicked problems *(e.g. How can we achieve sustainable mobility?)*
- specific papers with rather concrete real-world application *(e.g. Tipping points in open systems: bifurcation, noise-induced and rate-dependent examples in the climate system)*
- specific mathematical topics (relevant for sustainability) going slightly beyond the mathematical content of the course *(e.g. Robust optimization)*

Target group

This time:

- 24 students
- mostly Master of education with math as one of their subjects; some students math Bachelor

In general

- prerequisities: Linear Algebra, Analysis; maybe some form of Probability Theory / Statistics
- already first experiences with understanding and writing (advanced) math beyond basic exercises

Goal

- For Bachelor students: see various directions of math, also to form opinion for their future Master courses
- For Master of Education: motivation to study advanced math, important connections to sustainability for their future teaching job

Didactical perspective

- different ways to convey knowledge and stimulate the students
- special motivation due to connections to sustainability and advanced math

Group work

- fixed groups of 3 people
- applying light version of EduScrum: well-structured group work
- initialization of group work with reflection on strengths and weaknesses

Testing

- challenges ensure some level of mathematical rigor
- notes on Miro-Board and intermediate presentations help to ensure progress

For final test, constructive alignment is difficult:

- There is a **range of skills** to acquire.
- The general topics are **broad** and the students are allowed to choose **specializations** rather freely.
- It is always difficult to test understanding, knowledge, technical skills; here it is particularly challenging due to the breadth

Format of final test

- oral group exam: 10 mins talk as group, then questions on talk and on general lecture content
- written report: about 2500 words; short summary of what math was learnt beyond the course connected with the course content; what was learnt in the context of sustainability; relection about advances as mathematician and beyond

Obstacles

- difficult to match the **formal frame** of a normal course: schedule and exam
- **not much time** per subject area
- properly understanding real-world questions requires **advanced knowledge** and techniques
- all has to be **well aligned** and timed
  (difficult with public holidays, absences)
- advanced **course structure** (one has to get used to this)

Feedback and Conclusions

To be improved . . .

- **switching topics** so quickly is hard to digest

$\Rightarrow$ fair point, and not every course should be like this

- **proper supervision** and support for the project work is crucial

$\Rightarrow$ better instructions for supervisors; improved descriptions for project questions

- **sufficient resources** for missed lectures helpful

$\Rightarrow$ create lecture notes; better overview of further references

Feedback and Conclusions

To be expanded . . .

- **motivation** due to connections (including sustainability)

$\Rightarrow$ include more further connections in general

- it was helpful to learn structured **group work** and to be forced to work on several projects together

$\Rightarrow$ more guided group work in other courses, groups also inspired students to re-think their learning strategies

- new **strategies** to deal with obstacles: estimate ones boundaries, more endurance due to motivating setting

$\Rightarrow$ include more motivation beyond grades also in other courses

- increased motivation due to less pressure due to **more flexibility** and also due to group as support network

$\Rightarrow$ rethink distribution of work load in other courses

Further development:

- varying the level of abstraction
  - **simplifications** for a less mathematically educated audience
  - expanding on the more **abstract** mathematical theories lying behind the above mentioned fields (*algebra (solving systems of equations, theory of differential equations), topology (fixed points, solution sets), number theory (discrete optimization), (numerical / complex) analysis*)
- better coordination of
  - input (lecture): **proper lecture notes**, more references
  - material (project): small **explanations for all topics** so that it is easier to get an overview and have a quicker entrance to the mathematical questions
  - supervision: more **detailed feedback**, **proper hints** for the topics

Further details: structure of the lectures

- per subject area: 1st lecture basic, 2nd lecture advanced (additional lecture for data & probability)
- slides + blackboard, (nearly) no proofs, several examples and illustrations
- relation to sustainability topics mentioned throughout but only superficial
- carefully chosen reference lists (about 5 books per topic)

Conclusion

- potential of the content for other audiences and research
- potential of the structure for other courses