

Teaching AI for Math

Georg Loho

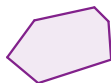
FU Berlin

28. Juli 2026

Potsdam

About Me

- ▶ Applications of polyhedral geometry: (discrete) optimization, machine learning (expressivity, explainability, learning structure), game theory, combinatorics



- ▶ Computer as tool
 - ▶ `polymake` developer (mostly during my PhD)
 - ▶ Using Oscar, Sage, ... in research process
 - ▶ Experimenting with use of LLMs
 - ▶ Basics of polyhedral geometry in Lean

Workshop: 'Polyhedra in Lean' (August 24 – September 4, 2026 at FU Berlin)

Idea of the course

- ▶ Learn mathematics
- ▶ Learn how to use computers
- ▶ Get a feeling for the power of AI

Conceptual blocks

- ▶ Mathematics as language: formalization, constructive mathematics, set theory vs type theory
- ▶ Reasoning: (automated) logical deduction
- ▶ Computing objects for existence and counterexamples: use 'old' mathematical software like SAT solvers, MIP solvers, Groebner bases computations, ...
- ▶ Generating and learning from mathematical data
- ▶ Finding mathematical objects and proofs as a game: reinforcement learning
- ▶ LLMs: N-gram language models for intuition, experimenting with most recent models

Structure of unit

- ▶ Introduction to the method
- ▶ historical or recent success stories
- ▶ show how to apply to 'new' problem

Mathematics as (formal) language: Lean

```
namespace Convexity

variable {R X Y V A : Type*}

open ConvexSpace

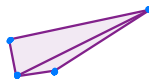
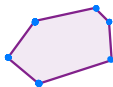
section Semiring

variable [Semiring R] [PartialOrder R] [IsStrictOrderedRing R]
variable [ConvexSpace R X]

variable (R) in
/-- A set is a polytope if it is the convex hull of finitely many points. -/
def IsPolytope (s : Set X) : Prop :=  $\exists t : \text{Finset } X, s = \text{convexHull } R t$ 

end Semiring
```

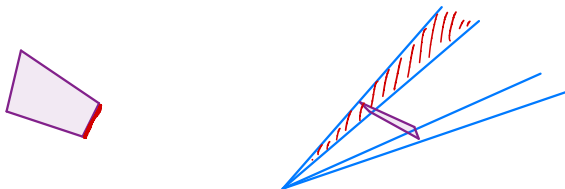
<https://github.com/ooovi/Polyhedral/blob/main/Polyhedral/Mathlib/Geometry/Convex/ConvexSpace/Polytope/Basic.lean>



Mathematics as (formal) language: Lean

```
/-- Faces of polytopes are polytopes. -/  
theorem face_isPolytope (hCfg : IsPolytope R (C : Set A)) (F : C.Face) :  
  IsPolytope R (F : Set A) := by  
  letI hom : IsHomogenization R A (CanonicalHomogenization R A) := inferInstance  
  letI := IsModuleConvexSpace.ofAddTorsor (R := R) (V := (CanonicalHomogenization R A))  
  have homC := IsPolytope.homogenize_FG (W := (CanonicalHomogenization R A)) hCfg  
  have homF := hom.homogenize_isFaceOf F.isFaceOf  
  have := PointedCone.IsFaceOf.fg homC homF  
  convert FG.dehomogenize_isPolytope this (fun _ a b ↦ weight_pos_of_mem_homogenize a b)  
  simp [dehomogenize_homogenize]
```

<https://github.com/ooovi/Polyhedral/blob/main/Polyhedral/Mathlib/Geometry/Convex/ConvexSpace/Polytope/Face.lean>



Further take-away: mathlib as **the** source of mathematical basics

Constructive mathematics: proving vs computing

Example: There exist irrational numbers x and y such that x^y is rational.

(Dov Jarden, 1953)

Non-Constructive

Constructive

Constructive mathematics: proving vs computing

Example: There exist irrational numbers x and y such that x^y is rational.

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Non-Constructive

consider $\sqrt{2}^{\sqrt{2}}$

$\sqrt{2}$

Constructive

Consider $\sqrt{2}$ and $\log_2(9)$

Constructive mathematics: proving vs computing

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Non-Constructive

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Consider $\sqrt{2}$ and $\log_2(9)$

- Further reading: Constructive Mathematics and Computer Programming (Martin-Löf 1982)

General deduction

[Home](#) > [Journal of Automated Reasoning](#) > [Article](#)

A Deductive Database Approach to Automated Geometry Theorem Proving and Discovering

Published: October 2000
Volume 25, pages 219–246 (2000) [Cite this article](#)

 [Save article](#)

Shang-Ching Chou, Xiao-Shan Gao & Jing-Zhong Zhang

 1093 Accesses  78 Citations  1 Altmetric [Explore all metrics](#) →

<https://doi.org/10.1023/A:1006171315513>

Article | [Open access](#) | Published: 17 January 2024

Solving olympiad geometry without human demonstrations

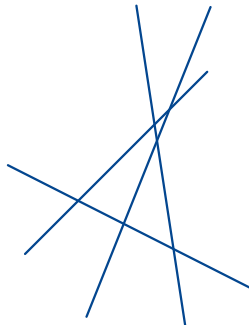
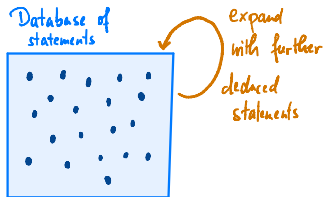
[Trieu H. Trinh](#) , [Yuhuai Wu](#), [Quoc V. Le](#), [He He](#) & [Thang Luong](#) 

[Nature](#) 625, 476–482 (2024) | [Cite this article](#)

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<https://doi.org/10.1038/s41586-023-06747-5>



SAT solvers and LLMs

► Counterexamples to two conjectures about matroids

(Matt Larson, 2026) <https://arxiv.org/abs/2607.02208>

Idea of one of the conjectures: strong symmetric basis exchange properties for pairs of bases

How Example 1.4 was found. After checking that there are no counterexamples to Conjecture 1.2 in rank at most 4, I prompted ChatGPT 5.5 Pro to try to check Conjecture 1.2 in small rank using a SAT solver. It implemented an approach and claims to have used this to prove Conjecture 1.2 in rank 5 and 6¹. I checked Conjecture 1.2 for many random matroids of ranks 7, 8, 9, and 10 which are realizable over $\mathbb{F}_2, \mathbb{F}_3, \mathbb{F}_4$, or $\mathbb{F}_5$. I prompted ChatGPT to look for a counterexample around 20 times; see [here](#) for a typical chat. I then prompted it to look for ternary matroids of ranks 7, 8, 9, and 10 and then to look for binary matroids of ranks 7 and 8. When I prompted it to look at binary matroids of rank 9, it found a matroid isomorphic to the matroid in Example 1.4; see [here](#).

Another description of the matroid in Example 1.4 is that it is isomorphic to the binary matroid realized by the row span of the 9×18 matrix $[I|A]$, where I is the 9×9 identity matrix and

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

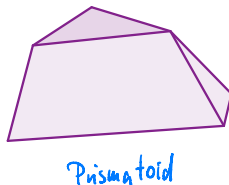
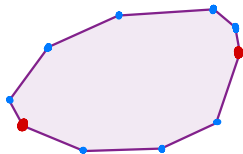
¹I have not verified the correctness of this computation.

SAT solvers and LLMs

- ▶ Recently: counterexample to 'Jacobian Conjecture'
 - ▶ Alpöge (2026) with Fable: https://x.com/__alpoge__/status/2079028340955197566
 - ▶ Zihan Zhang (2026) some mathematical implications: <https://zzhang-iu.github.io/papers/direct-consequences-jacobian/>

Polyhedral computations and LLMs

- ▶ A counterexample to the Hirsch Conjecture
(Paco Santos, 2012; Annals of Mathematics)
the graph of a d -dimensional polytope with n facets **can** have a diameter greater than $n - d$:
explicit construction of a 43-dimensional polytope with 86 facets and diameter at least 44.



Polyhedral computations and LLMs

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the graph of a d -dimensional polytope with n facets **can** have a diameter greater than $n - d$:
explicit construction of a 43-dimensional polytope with 86 facets and diameter at least 44.
- ▶ Advancing Geometry with AI: Multi-agent Generation of Polytopes <https://arxiv.org/abs/2502.05199>
Drums of high width
<https://londmathsoc.onlinelibrary.wiley.com/doi/abs/10.1112/plms.70031>

Take aways from earlier versions of the seminar

- ▶ Well-prepared technical setup important for students to actually use the tools
- ▶ Different aspects of difficulty: mathematical understanding of the specific questions, technical affinity, combining different approaches

Recap conceptual blocks

- ▶ Mathematics as **language**: formalization, constructive mathematics, set theory vs type theory
- ▶ Reasoning: (automated) logical **deduction**
- ▶ **Computing objects** for existence and counterexamples: use 'old' mathematical software like SAT solvers, MIP solvers, Groebner bases computations, ...
- ▶ Generating and learning from **mathematical data**
- ▶ Finding mathematical objects and **proofs as a game**: reinforcement learning
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Further considerations

- ▶ Ethics
- ▶ Social aspects
- ▶ Sustainability